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ÉCOLE DES HAUTES ÉTUDES COMMERCIALES
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CENTRE DE RECHERCHE SUR LES TRANSPORTS
UNIVERSITÉ DE MONTRÉAL
C.P. 6128, SUCCURSALE A
MONTRÉAL
CANADA H3C 3J7

TÉLÉPHONE : (514) 343-7575
EMAIL : crt@crt.umontreal.ca
TÉLÉCOPIEUR (FAX) : (514) 343-7121

CRT-837

LINEAR OR NONLINEAR UTILITY FUNCTIONS IN LOGIT MODELS? THE IMPACT ON GERMAN HIGH SPEED RAIL DEMAND FORECASTS.

by

Benedikt Mandel[1,2]
Marc Gaudry[1]
Werner Rothengatter[2]

[1] Centre de recherche sur les transports, Université de Montréal, C.P. 6128, succursale A, Montréal, Canada H3C 3J7.
[2] Institut für Wirtschaftspolitik und Wirtschaftsforschung, der Universität Karlsruhe (TH), Kollegium am Schloβ, Bau IV, Postfach 6980, D-7500 Karlsruhe 1, Fed. Rep. of Germany.

Centre de recherche sur les transports — Publication #837
Département de sciences économiques — Cahier de recherche #9214
Institut für Wirtschaftspolitik und Wirtschaftsforschung — Veröffentlichung #1/6/1992

June 1992

Dépôt légal – 2e trimestre 1992

93-01801/

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ABSTRACT

We have shown that nonlinearity of the representative utility functions of the Logit model applied to an intercity mode choice model implied the presence of asymmetry and of thresholds in the response curve, as well as results that had more reasonable properties (diminishing marginal values of time savings, elasticities and values of time that differed among the modes), and yielded very different high speed rail market shares for Germany than results obtained with the usual linear utility functions.

Keywords: BOX-COX LOGIT, HIGH SPEED RAIL, GERMANY, FORECASTS, VALUE OF TIME.

ZUSAMMENFASSUNG

Wir haben gezeigt, daβ sich die Existenz von Asymmetrie und Schwellenwerten in der Reaktionskurve mittels einer nichtlinearen repräsentativen Nutzenfunktion eines Logit Modells für ein Intercity Modalwahlmodell nachweisen läβt. Die dabei erzielten Ergebnisse basieren auf besseren realitätsnahen Modelleigenschaften ("diminishing marginal values of time savings", verkehrsträgerspezifische Elastizitäten und "values of time") gegenüber den linearen Modellen und die prognostizierten Marktanteile des Verkehrsträgers Schiene in der Bundesrepublik Deutschland differieren in signifikanter Weise von denen, die mittels gebräuchlichen linearen Nutzenfunktionen erzeugt werden.

Stichworte: BOX-COX LOGIT, SPURGEFÜHRTER HOCHGESCHWINDIGKEITS-VERKEHR, BUNDESREPUBLIK DEUTSCHLAND, PROGNOSEN, "VALUE OF TIME".

RÉSUMÉ

Nous avons montré que la présence de non-linéarités dans les fonctions d'utilité représentative des modèles Logit appliqués à un problème de choix entre modes de transport interurbains établissait clairement que la fonction de réaction des consommateurs comportait un seuil et était asymétrique. Nous avons par ailleurs trouvé des résultats plus raisonnables (utilité marginale décroissante des économies de temps, élasticités et valeur du temps de transport qui varient entre les modes) que les résultats linéaires et qui impliquent des prévisions de parts de marché des trains à grande vitesse en Allemagne qui diffèrent considérablement des résultats obtenus avec les fonctions d'utilité linéaires habituelles.

Mots-clés: BOX-COX LOGIT, TRAINS à GRANDE VITESSE, ALLEMAGNE, PRÉVISIONS, VALEUR DU TEMPS.

INTRODUCTION*

In many areas, one is confronted with discrete choice problems, where one chooses an alternative out of a set of available alternatives. To illustrate this problem in a simple way, just imagine the daily problem of shopping. Usually one has to select one product (for example mineral water) out of many alternative products. Although in principle these products are satisfying the same necessity, the consumer choice varies significantly among these alternatives. So if one assumes that each consumer of given socioeconomic characteristics or constraints will choose the alternative with the highest utility, and that the value of the utility itself is based on a comparison of the different alternative specific characteristics which describe the products (for example: price, quantity, mineral ingredients, carbon dioxide, etc.), the consumer's discrete choice can be modeled. The question which naturally arises then is whether different amounts of the characteristics of the alternatives, or different levels of the socioeconomic variables make contributions to utility that, for each unit, vary with the level considered. More formally, is this process linear or nonlinear?

Using the example of the "transportation mode choice on long distances", where the different carriers provide the set of alternatives or products, we want to discuss the choice problem with particular emphasis on the adequate representation of significant changes in transport conditions (fares, service levels). Although this is one of the most difficult modelling tasks, it is crucial to the evaluation of the profitability of many transportation investments.

In this paper, we compare high speed rail mode choice forecasts for Germany obtained from two variants of a mode choice model constructed with utility functions that are rich in both network and socioeconomic variables. In that sense, the chosen disaggregate specification, called MK-II, is representative of the state-of-art when refined networks pertaining to many modes (plane, train, car in this case) and large samples (62.982 observations in this case) are available. In Table 1, the variables used in the model specifications are shown by category.

The socioeconomic variables and the trip purposes, as well as the origin-destination pairs, were given by the KONTIFERN database which was developed under the auspices of the German Ministry of Transportation in the years 1979/1980. The data had been checked for consistency, validity and completness and obvious outlyers had been eliminated.

To enrich the database with modal variables, network characteristics for each of the three possible modes were developed for the year 1979/1980 and 1985. For each mode, these characteristics were obtained with shortest path algorithms, which take into

* This work is based on research directly financed by the German Ministry of Transportation and benefited indirectly from Marc Gaudry's tenure as an Alexander von Humboldt research prize awardee, as well as from Transport Canada and from National Sciences and Engineering Research Council of Canada funding.

account several link dimensions like transfer time, minimum connecting time and car driving pauses.

Table 1. Variables of the MK-II Model Specification

Mode specific variables	Socioeconomic variables	Trip purpose
travel cost	age class 0 to 9 years	business
travel time	age class 10 to 14 years	private
frequency	age class 15 to 17 years	[vacation]
trip distance *	age class 18 to 24 years	
access/egress distance	age class 25 to 44 years	
to the airport *	age class 45 to 64 years	
	[age class 65 years or more]	
	sex: male	
	employee	
* Variable used only for the adjustment of the MK-II L - and MK-II B models		

1. THE LINEAR AND BOX-COX LOGIT VARIANTS OF MODEL MK-II

Both variants of the MK-II model are based on the Logit model type and so the probability that individual n chooses alternative i out of a set of available alternatives C_n can be expressed as:

$$P(i)_n = \frac{\exp(V_{in})}{\sum_{j \epsilon C_n} \exp(V_{jn})}$$

where V_{jn} denotes the representative utility of the j^{th} mode for individual n. For the linear Logit model type, V_{jn} is usually defined as an additive combination of K linear variables X_{kjn} (i.e. characteristics), such as:

$$V_{jn} = \sum_{k=1}^{K} \beta_k X_{kjn}$$

The nonlinearity is introduced in the Box-Cox Logit model type by transforming one or more strictly positive variables X_{kjn} of the function V_{jn} into nonlinear variables $X_{kjn}^{(\lambda_k)}$ according to:

$$X_{kjn}^{(\lambda_k)} = \begin{cases} \left(X_{kjn}^{\lambda_k} - 1\right)/\lambda_k & \text{if } \lambda_k \neq 0 \\ \ln X_{kjn} & \text{if } \lambda_k = 0 \end{cases}$$

All parameters can be estimated by the usual maximum likelihood regression method.

The first model variant used, MK-II L, is a linear Logit model type. This form has the usual properties, such as:

i) the effect of a given difference in transport conditions is independent of the service level: therefore a 20 minute train service improvement has the same impact on choice probabilities for a short 40 minute trip and a long 400 minute trip. Similarly adding an amount of $ 10 to the price of traveling by plane will have the same impact as adding $ 10 to the price of traveling by train;

ii) the equal cross elasticities of demand imply that setting up a bicycle path between two cities will draw the same percentage of travelers from the modes car, train and plane;

iii) the values of time are identical across the modes so that representative train and plane users value time identically;

iv) the sigmoid shaped response curve shown in Figure 1 is symmetric with respect to its inflexion point, where modal shares are equal. This implies that, for example, asymmetry of behavior, where consumers/travelers suddenly start to react and then change their behavior, cannot be detected, even if it is crucial for the planing of transport investments.

The second variant, MK-II B, is a Box-Cox Logit model type (Gaudry and Wills, 1978). It dominates the first in a statistical sense and also yields values that are more reasonable in many ways. For instance, as the optimal power transformation on the modal variables is of the order of $\lambda = 0.24$ (a value that differs significantly from both 0 (the logarithmic case) and 1 (the linear case)), the impact of time or fare changes decreases with trip time or cost (diminishing returns); moreover, values of time vary across modes and depend on how much time is saved. Of course, we use the expression "value of time" conventionally here: in mode choice models, the total number of trips by all modes is assumed to be given. The computed trade-off, or marginal rate of substitution between time and money, is consequently a conditional value.

The optimal modal diversion curve revealed by the data is clearly not symmetric in its response to changes in travel time or fares. In Table 2, some of the results mentioned before are summarized. They were obtained with the "trio software" (Gaudry *et al.*, 1992) and carefully checked to insure that a global maximum had been found.

To illustrate the difference between the linearity and nonlinearity of a variable, Figure 1 shows the impact of the power transformation. The asymmetric curve (in respect to its inflexion point) given by the Box-Cox transformation of the strictly positive variable X_1 illustrates the error which will occur when a nonlinear variable is forced to be linear. For example, assume X_1 is the change of the total travel time: in the linear case, the value X_1 equal to 4 is associated to the probability P_1 equal to 0.25; in the nonlinear case, if $\lambda < 1$ the probability is higher and if $\lambda > 1$ the probability is smaller. Hence, if one forces a nonlinear variable to be linear this will result in an over- or underestimation of the probability related to this variable. Furthermore, in

addition to asymmetry of the response function ($\lambda \neq 1$), thresholds (where people start or stop reacting) can be identified.

Table 2. Comparison of Linear and Box-Cox Logit Results

Models for Germany, 1979/1980:			Linear Logit MK-II L	Box-Cox Logit MK-II B
Characteristics:				
• Own elasticity*	- travel costs	plane	-0.994	-0.624
		train	-0.131	-0.246
		car	-0.040	-0.043
	- travel time	plane	-0.753	-1.629
		train	-0.639	-1.006
		car	-0.081	-0.145
	- frequency	plane	0.122	0.109
		train	0.198	0.084
• Value of time (DM/minute)**		plane	0.932	3.183
		train	0.932	0.731
		car	0.932	1.092
• Power transformation λ on modal variables			1.000	0.240
• Final Log-Likelihood value			-1230.664	-1189.347
• Rho-squared bar of Hensher and Johnson			0.259	0.284

* Results for socioeconomic and trip purpose variables are not shown.
** This value includes time and other time related service components.

A more formal statement of the Box-Cox properties is given in Table 3. The Box-Cox transformation of the strictly positive variables of the linear Logit model leads to the Box-Cox Logit model with an asymmetry of response, as shown in Figure 1, because the effect of a unit change in the service will depend on the level of the variables X_{jk} for all values of λ_{jk} not equal 1. To see how a unitary improvement in one mode will have this effect, the partial derivatives of the representative utility V_j of the j^{th} mode used in the MK-II models are given in Table 3.

It can be seen clearly that the effect of additional service will be smaller at higher service levels than at lower ones if λ_{jk} is smaller than 1. These diminishing returns mean that given absolute reductions in total travel time have more impact when total travel times are low than high. Conversely, increasing returns exist if λ_{jk} is larger than 1. Hence, a λ smaller than 1, as that estimated in the MK-II Box-Cox Logit model, tells us that a gain of 10 minutes means less on a long trip than on a short one. For more details see Gaudry (1992), Mandel (1992) and forthcoming paper by Mandel *et al.* (1992).

Figure 1. Linear Logit versus Box-Cox Logit

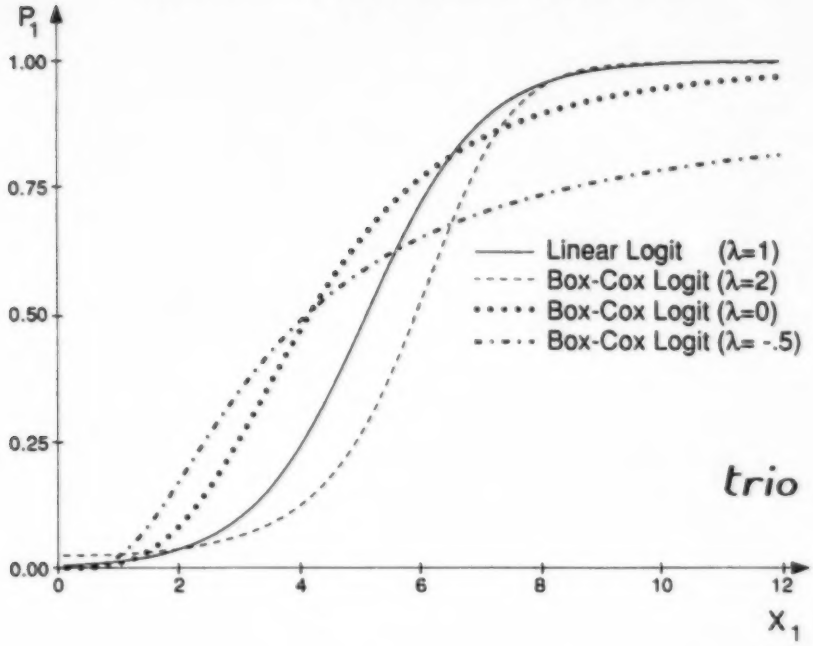

Table 3. Non Constant Returns in a Box-Cox Logit Model

Partial derivatives:		$\partial V_j/\partial X_{jk} = \beta_{jk} X_{jk}^{(\lambda_{jk}-1)}$	$\partial V_j^2/\partial X_{jk}^2 = \beta_{jk}(\lambda_{jk}-1) X_{jk}^{(\lambda_{jk}-2)}$
Returns:			
• Decreasing	$\lambda = -1$	β_{jk}/X_{jk}^2	$-2\beta_{jk}/X_{jk}^3$
	$\lambda = 0$	β_{jk}/X_{jk}	$-\beta_{jk}/X_{jk}^2$
• Constant	$\lambda = 1$	β_{jk}	0
• Increasing	$\lambda = 2$	$\beta_{jk} X_{jk}$	β_{jk}

2. FORECASTING

2.1. Adjustment to Observed Aggregate Flows of 1985

Formally, the forecasting exercise involves two distinct steps. When the parameter values for the two MK-II variants, derived from the 1979 (disaggregate) survey, are used to predict 1985 market share values, it is clear that they differ from "true" values for that year.

This can be understood if the 1979/1980 survey data systematically underreport all trips, and specially trips of shorter distances as may occur if forgetfulness yields a

systematic bias in the reporting of trips by distance. Furthermore, the aggregate flow matrices of 1985 are considered very reliable because the base of this survey is given by simulation processes which are controlled by direct observations for the car mode, by the tickets sold by the German Bundesbahn for the train mode and by the public air transportation statistics and internal statistics of Lufthansa for the plane mode. The possible bias in this set of data is assumed to be small although, to complete the matrix, the traffic flows between some origin-destination pairs had to be estimated or computed by simulation processes. The necessary modal variables for the MK-II models had been added to the 1985 survey data by using the shortest path algorithms on networks developed for that year.

The biases mentioned above were reflected in the difference between the forecasted and the "true" values, which was strongly and systematically related to the travel distance and differed across the modes. In a first step, correction factors are therefore incorporated with parameter values that are in keeping with the functional form of each variant.

The new utility functions V_{jn}^{*} include the complete old utility functions V_{jn} as well as the variable trip distance for the alternatives car and train and the variable access/egress distance to the airport for the plane alternative. If there is no difference between the forecasted and "true" values the new utility functions V_{jn}^{*} will be equal to the old utility functions V_{jn}. Moreover, Box-Cox transformations on the distance variable were used to examine whether the difference between the forecasted and the "true" values mentioned above is related to this variable in a nonlinear way or not.

We found again that power functions give a much better adjustment than linearity of the distance correction factor: this implies that the 1979 survey data systematically underreport all trips, but in a way that decreases rapidly with the distance (trip length) as the value of 0.52 shown for the λ in Table 4 requires. An incomplete comparison of the linear and nonlinear adjustments applied to the distance variables of the Linear and Box-Cox Logit models is given in Table 4.

Table 4. Comparison of the Linear and Nonlinear Adjustments of the MK-II Models

Models for Germany, 1985:	Linear Logit MK-II L		Box-Cox Logit MK-II B	
Characteristics:	Linear	Nonlinear	Linear	Nonlinear
• Coefficient of V_j	1.016	1.013	0.999	0.995
• Power transformation λ*	1.000	3.482	1.000	0.521
• Final Log-Likelihood	-1795.683	-1639.144	-15201.524	-15184.223
• Rho-squared bar	0.314	0.313	0.492	0.514
* Applied to distance variable				

2.2. Results of the Linear and Box-Cox Logit MK-II Variants

In the second step, the variants, "augmented" with functional correction factors, are used to forecast market shares for various scenarios involving major changes in travel speeds in 2010. The reference scenario R and the planning scenario V differ only by three origin-destination high speed rail links, namely Hamburg-Berlin (286 km, 250/160 km/h, 127 minutes), Frankfurt-(Halle)Leipzig-Dresden ((383 km) 465 km, 160/200/250 km/h, (145 minutes) 173 minutes), and Nürnberg-Berlin (616 km 200/250 km/h, 226 minutes), which are only included in the scenario V. All other basic input data and constraints are the same for the linear Logit model with linear adjustment (MK-II L) and the Box-Cox Logit model with the nonlinear adjustment (MK-II B). For more details concerning the developed networks and databases used for the year 2010, see Mandel (1992).

2.2.1 General Differences

In Table 5 the major forecast results give the market share of the train for some regional origin-destination pairs which are highly influenced by the high speed rail links mentioned above. The size and geographical location of these regions is shown in Figure 2 by thick edges and numbers. Furthermore, the differences of the forecasts for the two scenarios R and V are noted according to model type: ΔL for MK-II L and ΔB for MK-II B. In the right-hand side column, the ratio of ΔB and ΔL is computed as an indicator of difference between the forecasts.

Obviously, Table 5 shows that the Box-Cox Logit variant MK-II B produces forecasted values that differ significantly from those of the linear Logit variant MK-II L, as the existence of an asymmetric modal response implies. In interpreting the results, one should remember that the forecasts for the different scenarios have been made under the *ceteris paribus* condition: the train is the only modified mode.*

A comparison of the forecasted shares shows that the linear results are higher than, or roughly equal to, the nonlinear ones if the distance between the origin-destination pair is quite low, or have a common regional border (Bayern-NFC SW / SE, Ext. a. Berlin-NFC N, Coast OFC-NFC N, Rhein/Main-NFC SW / SE). They also show that the difference increases with distance (Rhein/Ruhr-NFC SW / SE / N). Note also that the regions are large and no additional access/egress services, like Interregio trains, were included in scenario V. The very high increasing market shares are all related to origin-destination pairs with agglomeration points like Berlin, Hamburg, München, Dresden, etc..

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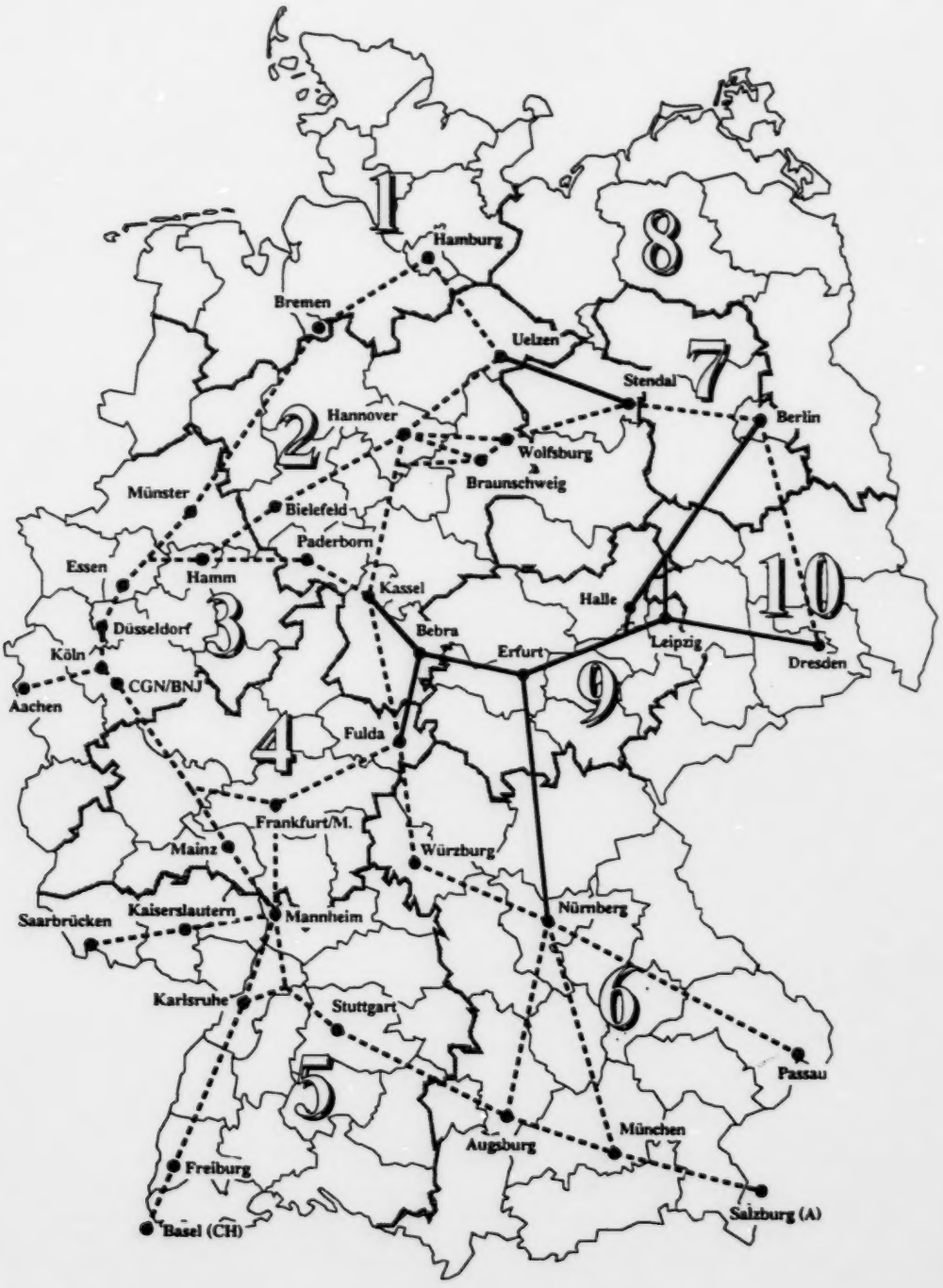
* It has to be remarked that it is not possible to compare the results shown in this paper directly with the results in Mandel (1992) because this paper uses an updated traffic flow matrix for the year 2010 and a slightly different demarcation of regions.

Table 5. Train Share Forecasts at the Regional Level for the Year 2010

No. - Regional origin - destination pairs		MK-II L			MK-II B			
		R	V	ΔL	R	V	ΔB	ΔB / ΔL
1 - Coast OFC	Ext. a. Berlin	10.7	15.1	4.4	11.6	18.0	6.4	1.45
	NFC N	9.5	9.6	0.1	7.6	7.8	0.2	2.00
	NFC SW	11.0	11.2	0.2	11.9	13.0	1.1	5.50
	NFC E	11.9	13.5	1.6	15.2	16.9	1.7	1.06
2 - North OFC	Ext. a. Berlin	15.7	16.1	0.4	18.8	19.7	0.9	2.25
	NFC N	10.6	11.1	0.5	11.8	12.7	0.9	1.80
	NFC SW	9.6	9.7	0.1	6.7	6.9	0.2	2.00
	NFC SE	10.4	12.0	1.6	11.3	14.1	2.8	1.75
3 - Rhein / Ruhr	Ext. a. Berlin	25.4	25.4	0.0	32.5	32.5	0.0	—
	NFC N	13.7	13.7	0.0	19.4	19.4	0.0	—
	NFC SW	11.4	12.6	1.2	12.6	14.7	2.1	1.75
	NFC SE	13.7	20.3	6.6	15.6	26.2	10.6	1.60
4 - Rhein / Main	Ext. a. Berlin	25.0	26.9	1.9	29.7	33.0	3.3	1.73
	NFC N	17.7	17.7	0.0	25.9	25.9	0.0	—
	NFC SW	9.4	10.5	1.1	8.1	10.4	2.3	2.09
	NFC SE	10.3	16.2	5.9	10.6	21.5	10.9	1.84
5 - Saar / Rhein /	Ext. a. Berlin	23.9	26.0	2.1	27.1	30.2	3.2	1.52
Neckar	NFC N	20.5	20.6	0.1	28.0	28.1	0.1	1.00
	NFC SW	11.2	13.0	1.8	12.5	15.9	3.4	1.88
	NFC SE	9.7	14.5	4.8	10.9	19.0	8.1	1.68
6 - Bayern	Ext. a. Berlin	12.0	18.7	6.7	13.0	23.5	10.5	1.56
	NFC N	11.7	13.6	1.9	16.0	18.8	2.8	1.47
	NFC SW	7.0	8.4	1.4	4.9	7.2	2.3	1.64
	NFC SE	6.0	7.0	1.0	3.9	5.4	1.5	1.50
7 - Ext. a. Berlin	NFC N	11.7	11.7	0.0	8.2	8.2	0.0	—
	NFC SW	9.6	13.4	3.8	6.5	10.0	3.5	0.92
	NFC SE	9.6	10.6	1.0	7.5	8.0	1.5	1.50
8 - NFC N	NFC SW	10.2	10.2	0.0	9.0	9.0	0.0	—
	NFC SE	9.3	9.3	0.0	9.6	9.6	0.0	—
9 - NFC SW	NFC SE - 10	9.0	9.8	0.8	5.1	6.1	1.0	1.25

OFC: Old Federal Countries NFC: New Federal Countries Ext. a.: Extended area
SW: South-West SE: South-East N: North

Figure 2. German High Speed Rail Network for the Year 2010

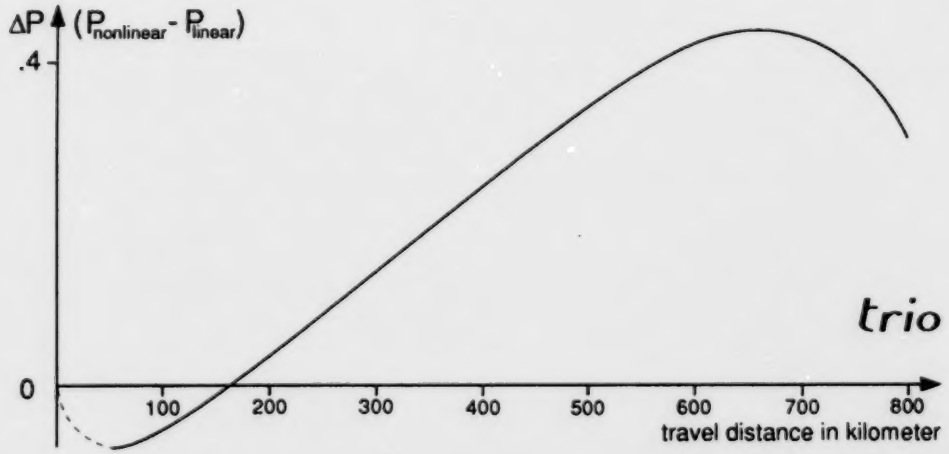


However, if we aggregate these individual values over the Federal Republic of Germany as a whole, we obtain a higher train market share for the linear than for the nonlinear model, which is logical because the number of travellers decreases with trip length.

The difference between the linear and nonlinear forecasts is also obvious if one looks at the ratio of ΔB and ΔL given in the last column of Table 5. On average, the impact of high speed rail links on the market behavior is predicted 1.78 times higher with the nonlinear MK-II B model than with the linear one.

This difference ΔP between the nonlinear and linear forecast is visualized in Figure 3 as an example which specially takes into account the new high speed rail links. The curvature varies in a small range for different origin - destination pairs but always keeps the shape shown so that it can be stated that the linear form tends to predict higher market shares on short distances than the nonlinear form, and the reverse for longer distances.

Figure 3. Difference of Nonlinear and Linear Forecasts Related to Travel Distance



2.2.2 Asymmetry

But, independently from the level of market shares, the difference (ΔL and ΔB) between the forecasts for scenario R and V is even more interesting from the perspective of asymmetry: do asymmetric modal response functions influence high speed rail demand forecasts?

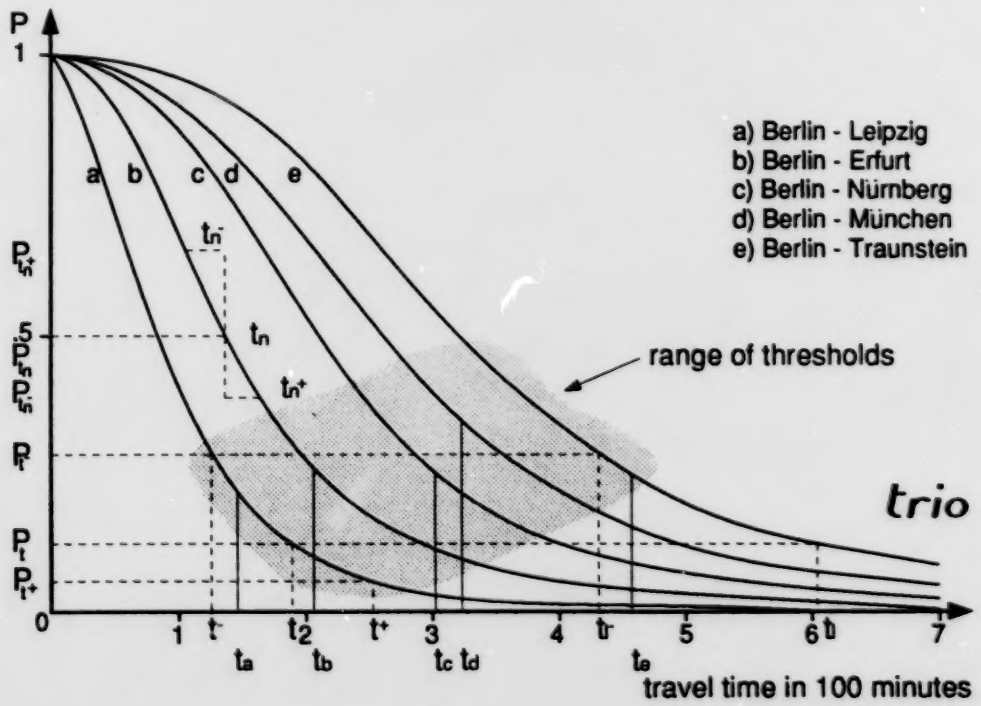
To illustrate the asymmetry of the response functions due to the inflexion point of the curve and the threshold effect mentioned before, Figure 4 shows for different origin-destination pairs the response curves with respect to the variable travel time for the train alternative while all other conditions (characteristics and modes) remain unchanged.

To describe asymmetry more formally one first has to define the inflexion point of the curve. At this point the curvature changes its functional shape from convex to concave and one can compute the value of the inflexion point (P_{t_n}, t_n) by equating to zero the second derivative of the MK-II B mode share in respect to the travel time. The response curve can be called asymmetric with respect to its inflexion point if equidistant reductions and increments of travel time t_n by Δt [that is, $t_n^+ = t_n + \Delta t$ and $t_n^- = t_n - \Delta t$] will give different absolute values, namely $\Delta P^+ = \left| P_{t_n} - P_{t_n^+} \right|$ and $\Delta P^- = \left| P_{t_n} - P_{t_n^-} \right|$: otherwise the curve has to be called symmetric. More formally, one can define asymmetry, as in Laferrière and Gaudry (1992) in terms of the partial correlation ξ of $P_{t_n^+}$ and $\left(1 - P_{t_n^-}\right)$: this yields an indicator that is necessarily between 0 and 1.

2.2.3 Thresholds

A threshold effect occurs when the travel time reaches a critical value of t beyond which any further reduction of t to $t^- = t - \Delta t$ provokes a more substantial growth of the mode share P_t than an equidistant increment of t to $t^+ = t + \Delta t$, so that the absolute difference of the mode shares $|P_t - P_{t^-}|$ is higher than $|P_t - P_{t^+}|$.

Figure 4. Asymmetry, Thresholds and the Impact of Travel Distance (MK-II B Model)



The word threshold implicitly involves an individual evaluation of the preception of change; hence it is up to the decision-maker to define his threshold by exploring the percentage of mode share increment which he will consider as a threshold i. e. which will satisfy his opinion about a threshold. More formally a critical value η has to be defined so that the absolute difference of $|P_t - P_{t-}| = (1 + \eta)|P_t - P_{t+}|$. Alternatively, $\Delta P_{t-} = (1 + \eta)\Delta P_{t+}$ and hence $\eta = (\Delta P_{t-}/\Delta P_{t+}) - 1$. From a visual point of view, one would intuitively expect to find the thresholds to be in the range given by the grey zone in Figure 4, where a reduction of one unit would increase the probability of choosing the alternative train by additional 10 to 50 percent ($\eta = 0.1$ to 0.5), so that ΔP_{t-} would be equal to 1.1 to 1.5 times ΔP_{t+}.

It is obvious that the given results in Figure 4 are based on a *ceteris paribus* assumption: consequently a variation of other mode specific characteristics like frequency or travel fare would imply a change of the location of the response curve so that the threshold would have to be relocated.

If one compares the different response curves in Figure 4 from left to right, the impact of travel distance on consumer behavior in respect to travel time can be seen. By increasing the distance, the asymmetric shape of the functions become more flat: this corresponds to the statement that a reduction of travel time by 15 minutes on short distances i. e. short trips has an higher impact on consumer behavior than on long trips. Conversely, one could say that to have the same impact on consumer behavior on long trips, as on short trips, one has to decrease the travel time by much more ($|t_l - t_{l-}| \ggg |t - t^-|$).

The t_α values marked on the x-axis represent the actual travel time on the different origin-destination pairs which are directly influenced by the high speed rail links mentioned before. One can say that on short distances and on long North-South origin-destination relations, the actual travel times are in the range of the thresholds, while on long West-East origin-destination relations an additional reduction of travel time would be necessary to catch the threshold effect.

It has to be mentioned that in general by interpreting the results shown in Figure 4 one has to take into account that the travel time represents the time of a door to door trip and that origin-destination regions (see thin edged areas in Figure 2) are relatively large, which means that the average access/egress time to the main mode is quite high. Therefore a change of the access/egress services can have an important impact on the choice of an alternative.

With respect to these points, one can detect in Table 5, by comparing a range of origin-destination pairs (Rhein/Main-NFC SW / SE, Rhein/Ruhr-NFC SW / SE, Bayern-SW / Ext. a. Berlin), a threshold related to distance, or total travel time, which has to be crossed so to obtain a strong change of demand. This market behavior is shown in column ΔL as well as in ΔB but, in the linear case, this threshold occurs at shorter distance than in the nonlinear one.

2.2.4 Other Considerations

The purpose of our example is to visualize the asymmetry of the response functions, the existence of thresholds and the impact of travel distance on consumer behavior: it is not to analyze in detail the different investment projects on high speed rail links mentioned before. If one wanted to do this, it would of course be necessary to consider in addition the impact of travel fare, frequency, access/egress characteristics, etc..

Moreover the formal statements made here on the topics of asymmetry and threshold are given for our particular example of a decreasing function (with respect to travel time) and have to be reversed if one examines characteristics like frequency, which imply an increasing reaction curve.

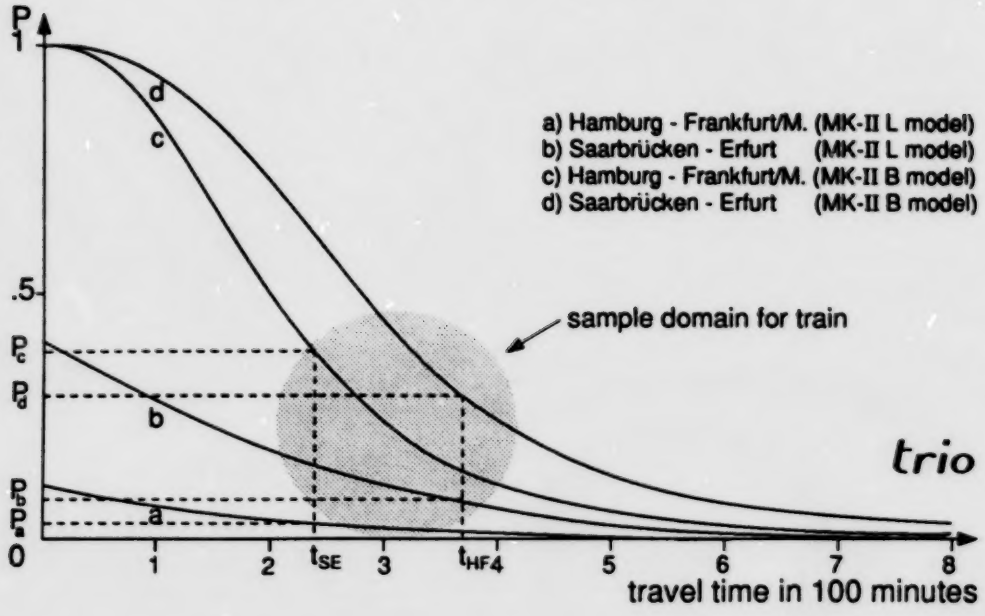
The examination shown in Figure 4 also can be done in reverse direction where one first defines the probability of choosing the mode and then computes the necessary characteristics which satisfy this condition. Different kinds of model services, which are related to different technologies like magnetic-, Intercity-express-, Pendolino-train/service, etc., are therefore represented by the underlying travel time. Implicitly we have the possibility to verify the optimal investment by relating it to the mode specific characteristics that maximize revenue.

2.2.5 Nonlinearity and Segmentation

One of the great benefits of nonlinearity is to reduce dramatically the need for certain kinds of segmentation — e. g. by fare or distance classes — typically done in order to obtain adequate representation or fit in domains other than that of the sample mean. Were one to forecast well outside of the sample domain (represented in Figure 5 by the standard derivation away from the mean), it is clear that a linear form would behave very differently from a nonlinear form. As high speed rail involves large changes in service levels, one would certainly not forecast with a linear form without segmenting the sample — in effect doing piecemeal linear approximation of the underlying nonlinear form.

In our model, the need for segmentation may still remain for different categories of trips — e. g. trip purposes. A convenient approach would consist in splitting the sample (if a dummy variable had indicated heterogenity of the full sample with respect to trip purpose) in order to see whether the curvatures differ by trip purpose or are adequately represented by a (constant) shift of the response curve obtained with the full sample.

Figure 5. Segmentation and the Sample Domain



3. THE IMPACT OF ASYMMETRY

Using linear and Box-Cox Logit variants of an mode choice model, we have shown that the presence of asymmetry of the reaction function implies quite different forecasts than the forecasts made with a linear form. In particular, the linear form tends to overpredict market shares for short distances and to underpredict market shares for longer distances. In particular, the influence of high speed rail links on consumer behavior is much stronger in the nonlinear case than in the linear one.

Furthermore we show that the existence of thresholds and asymmetric behavior of the response curve means that customers will significantly change their modal choice behavior if the threshold can be reached. Generally speaking, the nonlinear functional form found establishes that a 15 minute reduction of travel time for short trips has a much higher impact on mode choice than for long trips.

In addition, the benefits of nonlinearity can be seen in the different values of time and elasticities for each alternative (which are more reasonable than in the linear case) and in the reduction of the necessity to segment the sample in order to make credible forecasts involving significant changes in service attributes.

Because of the significant differences we found between the forecasts, we assume that this has impacts for the optimal sequence of investments.

ACKNOWLEDGEMENTS

This work is based on research directly financed by the German Ministry of Transportation and benefited indirectly from Marc Gaudry's tenure as an Alexander von Humboldt research prize awardee, as well as from Transport Canada and from National Sciences and Engineering Research Council of Canada funding.

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